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**Problem 19)** Let the horizontal axis be marked by integers  $1, 2, 3, \dots$ , which identify the individual sets  $S_1, S_2, S_3, \dots$ . Let the vertical axis also be marked by integers  $1, 2, 3, \dots$ , but these indices now identify the individual elements of each and every (countable) set. On the two-dimensional mesh thus created, the point  $(m, n)$  represents the  $n^{\text{th}}$  element of the  $m^{\text{th}}$  set. The set of points on this mesh can be counted similarly to those assigned to rational numbers  $m/n$ , depicted in figure 1.3. We conclude that the union of  $S_1, S_2, S_3, \dots$  represents a countable set.

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